Investigation

2 Connecting Ratios and Rates

In Investigation 1, you used fraction strips as a tool to determine the fraction of each fundraising goal reached and locate points and distances on a number line. You also used ratios to compare quantities and checked to see if they were equivalent. In this Investigation you will continue to explore ratios and ways to write equivalent ratios.

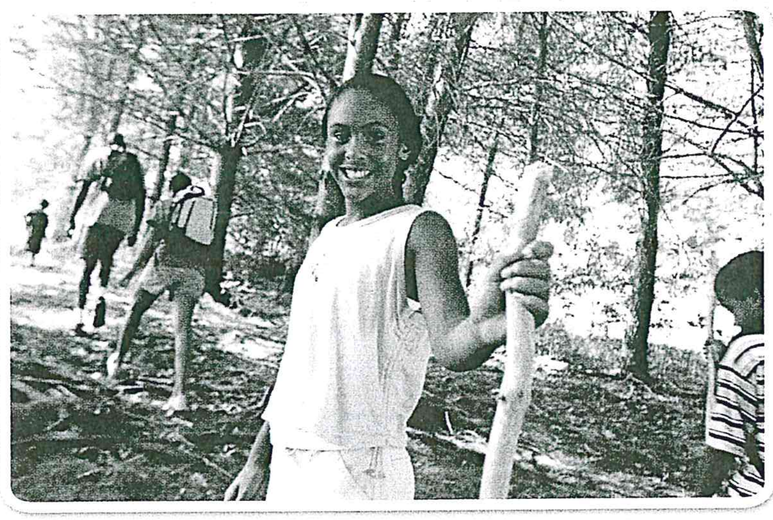
The ratio statements in Investigation 1 were written as "for every" or "to" statements. Ratios can be written in many different ways.

Suppose the cost for ten students to go on a field trip is $120. You can write ratios to show how the quantities are related.

10 students *for every* $120

10 students *to every* $120

10 students : $120



Common Core State Standards

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.A.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables . . .

6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed.

Also 6.RP.A.1, 6.RP.A.2, 6.NS.B.4

Ratio statements can also be written as "per" statements. For example, "It costs $120 per 10 students to go on the trip." An equivalent comparison statement is "the cost per student to go on a field trip is $12." Now you can say

$12 *for every* 1 student

$12 *for each* student

$12 *per* student

This particular comparison, cost per one student, is called a unit rate. A **unit rate** is a comparison in which one of the numbers being compared is 1 unit.

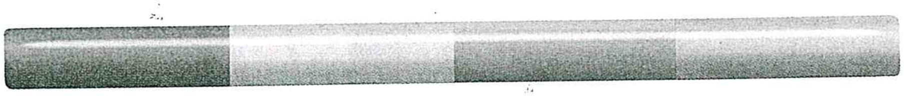
- If the cost of food is $250 for 50 students, what is the cost per student?

To answer this question, you find the unit rate.

2.1 Equal Shares
Introducing Unit Rates

Often we share food so that each person gets the same amount. This may mean that food is cut into smaller pieces. Think about how to share a chewy fruit worm that is already marked in equal-sized pieces.

The chewy fruit worm below shows four equal segments.



How can you share this 4-segment chewy fruit worm equally among four people?

How many segments of the worm does each person get?

OR

How can you share this 4-segment chewy fruit worm equally among three people?

How many segments of the worm does each person get?

Problem 2.1

In Questions A and B, find the fraction of a chewy fruit worm each person gets.

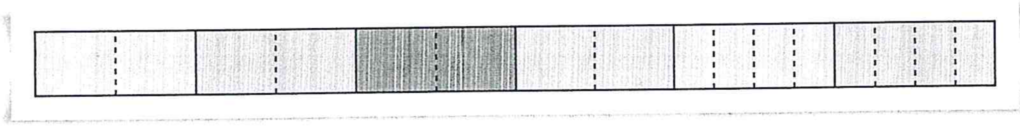
A 1. Show two ways that four people can share a 6-segment chewy fruit worm. In each case, how many segments does each person get?

2. Show two ways that six people can share an 8-segment chewy fruit worm. In each case, how many segments does each person get?

B 1. Show how 12 people can share an 8-segment chewy fruit worm. How many segments are there for every person?

2. Show how five people can share a 3-segment chewy fruit worm. How much is this per person?

C Jena wants to share a 6-segment chewy fruit worm. The tape diagram below shows the marks she made on the worm so she can share it equally among the members in her CMP group.



1. How many people are in her group?
2. Is there more than one possible answer to part (1)? Explain.
3. What is the number of segments per person?
4. Write a fraction to show the part of the chewy fruit worm each person gets.

D Would you rather be one of four people sharing a 6-segment chewy fruit worm or one of eight people sharing a 12-segment chewy fruit worm? Explain.

E Look back at your work on this Problem. Describe how you found or used unit rates.

ACE Homework starts on page 50.

2.2 Unequal Shares
Using Ratios and Fractions

Sometimes there are reasons to share quantities *unequally*. Suppose your older brother paid more than half the cost of a video game. You might think it is fair for him to spend more time playing the game. At a party, you might agree that your friend should take the bigger piece of chocolate cake because your friend likes chocolate more than you do.

Two sisters, Crystal and Alexa, are going to a strange birthday party. Instead of birthday cake, pairs of party guests are each served a large chewy fruit worm to share according to their ages. Since the sisters are not the same age, they do not share their fruit worm equally.

Crystal is 12 years old and Alexa is 6 years old. Their chewy fruit worm has 18 segments. According to their ages, Crystal gets 12 segments and Alexa gets 6 segments. The ratio of the girls' shares of the worm, 12 to 6, is equivalent to the ratio of their ages, 12 to 6.

- According to the rule, how would the girls share a 9-segment chewy fruit worm?

Since Crystal's age is two times Alexa's age, Crystal gets twice as many segments as Alexa. The ratio of Crystal's segments to Alexa's segments is 12 to 6 or 2 to 1.

- The ratio 2 to 1 is a unit rate. What do the numbers 2 and 1 mean for the sisters?

In this Problem you will explore situations that involve fractions and ratios.

Problem 2.2

A Draw some chewy fruit worms with different numbers of segments that Crystal and Alexa can share without having to make new cuts.

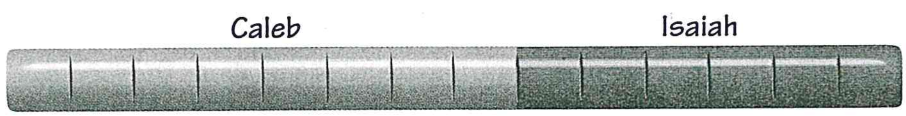
B **1.** Jared is 10 years old. His brother Peter is 15 years old. What are some chewy fruit worms they can share without having to make new cuts?

2. For each worm you described in part (1), write a ratio comparing the number of segments Jared gets to the number of segments Peter gets.

3. Are the ratios you wrote in part (2) equivalent to each other? Explain.

4. How would you write a unit rate to compare how many segments Jared and Peter get?

C **1.** Caleb and Isaiah are brothers. They share a 14-segment chewy fruit worm according to their age. How old could they be?



2. Caleb gets 8 out of the 14 segments of the chewy fruit worm, so he gets $\frac{8}{14}$ and Isaiah gets $\frac{6}{14}$ of the worm.

a. From Question A, what fractions of the chewy fruit worm do Crystal and Alexa each get at the birthday party?

b. From Question B, what fractions of the chewy fruit worm do Jared and Peter each get at the birthday party?

c. How does the ratio of segments that Caleb and Isaiah get relate to the fractions of the chewy fruit worm that they each get?

ACE Homework starts on page 50.

2.3 Making Comparisons With Rate Tables

When comparing how to share chewy fruit worms, Crystal recorded how many segments she and her sister would get for different sizes of chewy fruit worms. Crystal thought she could use what she knew about equivalence to make a table showing the amounts.

Comparing Segments

Segments for Alexa	6	3	1	2	$\frac{1}{2}$	10
Segments for Crystal	12	6	2	4	1	20

The table shows that for every segment given to Alexa, Crystal gets two segments. This is Alexa's unit rate. The table also shows that for every $\frac{1}{2}$ segment Alexa is given, Crystal gets one segment. This is Crystal's unit rate.

Crystal sees an ad for chewy fruit worms. She decides she wants the student council to include chewy fruit worms in the fundraising sale.

You can use the information in the advertisement to compute the price for any number of worms you want to buy. One way to figure out the price of a single item from a quantity price is use the information to build a **rate table** of equivalent ratios.

The rate table in Question A shows the price for different numbers of chewy fruit worms. The cost of 30 chewy fruit worms is $3.

Problem 2.3

A 1. Crystal wants to calculate costs quickly for many different numbers of chewy fruit worms. Copy and complete the rate table below with prices for each of the numbers of chewy fruit worms.

Chewy Fruit Worm Pricing

Number of Worms	1	5	10	15	30	90	150	180
Reduced Price	■	■	■	■	$3	■	■	■

2. How much do 3 chewy fruit worms cost? 300 chewy fruit worms?
3. How many chewy fruit worms can you buy for $50? For $10?
4. What is the unit price of one chewy fruit worm? What is the unit rate?

B The student council also decides to sell popcorn to raise money. One ounce of popcorn (unpopped) kernels yields 4 cups of popcorn. One serving is a bag of popcorn that holds 2 cups of popcorn.

1. Use a rate table to find the number of ounces of popcorn kernels needed to determine the cups of popcorn.

Cups of Popcorn From Ounces of Kernels

Number of Cups of Popcorn	4	■	■	■	■	■	■	■	■	■	■	■
Number of Ounces of Popcorn Kernels	1	2	3	4	5	6	7	8	9	10	11	12

2. How many cups of popcorn can you make from 12 ounces of popcorn kernels? From 30 ounces of popcorn kernels?
3. How many ounces of popcorn kernels are needed to make 40 cups of popcorn? To make 100 cups of popcorn?
4. How many ounces of kernels are needed to make 100 servings?
5. How many ounces of kernels are needed to make 1 cup?

C 1. How do rate tables help you answer Question A and Question B?
2. How do unit rates help you answer Question A and Question B?

ACE Homework starts on page 50.

Applications | Connections | Extensions

Applications

1. Show two ways three people can share a 5-segment chewy fruit worm.

2. Show two ways five people can share a 3-segment chewy fruit worm.

3. Sharon is ready to share the 4-segment chewy fruit worm shown below. She has already made the marks she needs so that she can share it equally among the members of her group.

   a. Give two different numbers of people that could be in Sharon's group.

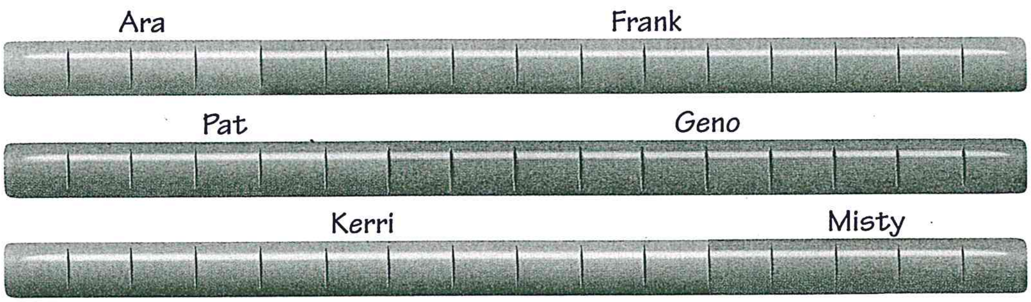
   b. For each answer you gave in part (a), write a ratio comparing the number of people sharing a chewy fruit worm to the number of segments they are sharing. How would you rewrite this as a unit rate?

4. Cheryl, Rita, and four of their friends go to a movie and share a 48-ounce bag of popcorn equally and three 48-inch licorice laces equally. Write a ratio comparing the number of ounces of popcorn to the number of friends. Then, write a unit rate comparing the length of licorice lace for each person.

5. The Lappans buy three large sandwiches to serve at a picnic. Nine people come to the picnic. Show three different ways to cut the sandwiches so that each person gets an equal share.

6. Three neighbors are sharing a rectangular strip of land for a garden. They divide the land into 24 equal-sized pieces. They each get the same amount of land. Write a ratio comparing the number of pieces of land to the number of people. Write the answer in more than one way.

7. For each chewy fruit worm below write the possible ages of the two people sharing the worm by age.



Use this information for Exercises 8–10. At the birthday party in Problem 2.2, the children run relay races. The distance each team member runs depends on the ratio of their ages. For example, a boy who is twice as old as a girl runs twice as far.

8. Crystal is 12 years old and Alexa is 6 years old. If Crystal runs 100 yards, how far does Alexa run? How far do they run altogether?

9. Jared is 10 years old and Peter is 15 years old. Together, they run 150 yards. How far does each brother run?

10. Wynne and Emmett are brother and sister. Wynne runs 180 yards. Emmett runs 120 yards. How old could each of them be?

Use this information for Exercises 11–14. Parents are older than their children. The ratio of a parent's age to a child's age changes as the parent and child get older.

11. Can a parent ever be exactly twice as old as his or her child? Explain.

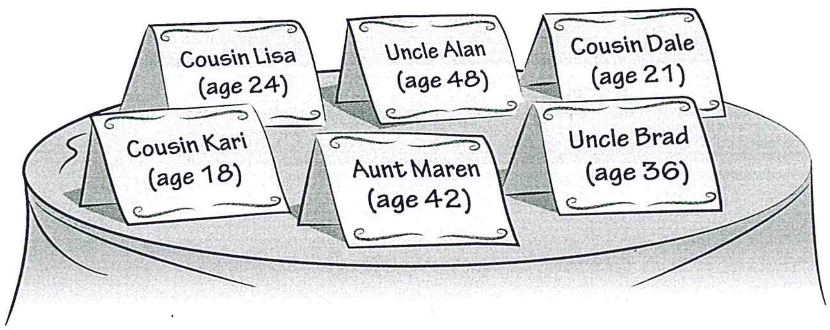
12. Can a parent ever be exactly three times as old as his or her child? Explain.

13. Can the ratio of a parent's age to his or her child's age ever be exactly 3 : 2? Explain.

14. Can the ratio of a parent's age to his or her child's age ever be exactly 10 : 9? Explain.

15. Crystal and Alexa convince the older members of their family to break up the chewy fruit worms using age ratios. They want to know which family members have the same age ratio as Crystal and Alexa.

a. Use the ages of their family members to find pairs that have the same age ratio as Crystal (age 12) and Alexa (age 6).



b. What do all the ratios that you wrote in part (a) have in common?

For Exercises 16–18, copy and complete the table comparing the chewy fruit worm segments each family member received. State both unit rates in each comparison.

16.

Segments for Alan	48	12	■	1	■	7
Segments for Lisa	24	■	8	■	1	■

17.

Segments for Lisa	24	12	■	1	■	■
Segments for Alexa	6	■	2	■	1	$1\frac{1}{2}$

18.

Segments for Alan	48	24	■	1	■	■
Segments for Alexa	6	■	2	■	1	$1\frac{1}{2}$

For Exercises 19–22, use the family members from Exercise 15, including Crystal and Alexa. Determine which two people have each age ratio.

19. The unit rate is 2 : 1.

20. The unit rate is 4 : 1.

21. The ratio of segments (ages) is 3 : 4.

22. The ratio of segments (ages) is 3 : 2.

For Exercises 23 and 24, Rosco is planning meals for his family. He uses the vertical rate tables.

23. **a.** Complete the rate table for the macaroni and cheese ingredients.

Macaroni and Cheese

Ounces of Macaroni	Cups of Cheese
8	1
▒	2
▒	3
▒	4
▒	5
▒	6

b. How many ounces of macaroni would you need for 7 cups of cheese?

c. How many cups of cheese would you need for 88 ounces of macaroni?

24. **a.** Complete the rate table for the spaghetti ingredients.

Spaghetti and Sauce

Ounces of Spaghetti	Ounces of Tomatoes
12	16
6	8
3	■
2	■
1	■

b. What is the unit rate comparing the number of ounces of tomatoes to 1 ounce of spaghetti?

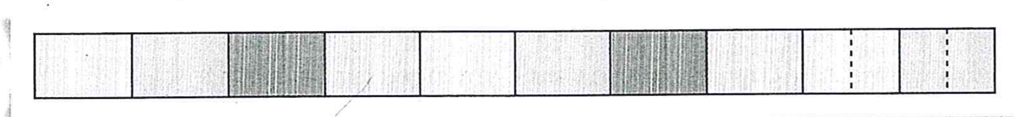
c. What is the unit rate comparing 1 ounce of tomatoes to the number of ounces of spaghetti?

Connections

25. Ursula, Ubaldo, Ulysses, and Dora were trying to come up with different ways to divide a 10-segment chewy fruit worm among the four of them. Which of these strategies would result in sharing equally?

- Ursula's Strategy:

  Give everyone two segments, and then divide the remaining two segments into four equal pieces with each person getting another half of a segment.



- Ubaldo's Strategy:

  Give each person one segment, then if there's at least four segments left, give each person another segment. Repeat this process until there are less than four segments, then cut the leftover pieces into four equal parts and give each person a part.

- Ulysses' Strategy:

  Give each person two segments, and then use a spinner to pick the winner of the extra two segments.

- Dora's Strategy:

  Forget about the segments. Just cut the worm in half, and then cut each half in half again.

26. If you were going to make segment marks on a chewy fruit worm without any marks, what would be the advantage or disadvantage of using a prime number of segments?

27. A typical container of orange juice concentrate holds 12 fluid ounces (fl oz). The standard recipe is "Mix one can of concentrate with three cans of cold water."

   a. What is the ratio of concentrate to water?

   b. How large of a container will you need to hold the juice?

   c. Olivia has a one-gallon container to fill with orange juice. She uses the standard recipe. How much concentrate does she need? (One gallon is 128 fl oz.)

28. A typical container of lemonade concentrate holds 12 fl oz. The standard recipe is "Mix one can of concentrate with $4\frac{1}{3}$ cans of cold water."

   a. What is the ratio of concentrate to water?

   b. How large of a container will you need to hold the lemonade?

   c. Olivia has a one-gallon container to fill with lemonade. She uses the standard recipe. How much concentrate does she need? (One gallon is 128 fl oz.)

29. Langhus Convenience Store sells multiple sizes of chewy fruit worms. Betsy, Emily, and John are trying to decide which of the deals would give them the most chewy fruit worms for the price.

a. Which argument do you think is the best? Explain.

- Betsy: The small size is the best deal because you get the most amount of worms, 10 more than the medium size, and 18 more than the large size.
- John: The large size is the best deal because you have to pay the least amount of money overall.
- Emily: I used the least common multiple of 4, 8, and 12, which is 24. For $24, I could buy 60 large worms, 54 medium worms, and 56 small worms. The large size is the best deal.

b. How could Betsy, John, and Emily use unit rates to find the best deal?

30. As Johann is working on unit rates in Exercises 16–24, he notices something interesting and says to his teacher, "Whenever you compare two quantities and you write both unit rates, at least one of them will have a fraction in it." Is Johann correct? Explain why you agree or disagree with him.

Extensions

For Exercises 31–33, consider the conjectures Jena made while working on Problem 2.1. Which conjectures do you think are true? Explain.

31. If the number of people is greater than the number of segments, each person will get less than one segment.

32. There are at least two ways to divide any chewy fruit worm so that everyone will get the same amount.

33. If the ratio of people to segments is 1 : 2, then each person will get $\frac{1}{2}$ of a segment.

34. Harold is eight years older than Maynard. On Harold's sixteenth birthday, he notices something interesting about their age ratios. He says, "When I was nine, the ratio of my age to Harold's was 9 : 1. A year later the ratio was 5 : 1. That's when I was ten and Maynard was two. Now on my sixteenth birthday, I'm twice as old as Maynard, which means the ratio of our ages is 2 : 1." Will Harold and Maynard ever have an age ratio 1 : 1? Explain.

35. A women's 4-by-100 meter medley relay team finished in second place. In the relay, each member swims 100 meters using a different stroke. The ages of the team members are 21, 22, 25, and 41.

The age difference between the oldest and youngest swimmer on this team was 20 years!

Suppose they had broken up the distance of 400 meters by age as in Problem 2.2. How far would each person swim in the relay?

36. Mariette, Melissa, and Michelle were given this follow-up question by Mr. Mirasola to Problem 2.3, "If you had $3.55, how many large chewy fruit worms could you buy?"

- Mariette said that she could buy $35\frac{1}{2}$.
- Melissa said that she could buy only 35.
- Michelle said that she could buy only 30.

Mr. Mirasola said, "You are all correct depending on how you think of the ad." How is it possible that they could all be correct?

37. On a recent trip to Canada, Tomas learned that there was an "exchange rate" between U.S. dollars and Canadian dollars. When he exchanged his U.S. dollars, he did not get the same number of Canadian dollars back. Tomas hopes to visit many different countries one day, so he does some research and finds a Web site with some basic money conversions on it.

a. Find the unit rate for each country below.

Currency Exchange Rates

$20 US ≈ 19 Australian Dollars	$1 US ≈ ■ AUD	$■ US ≈ 1 AUD
$5 US ≈ 4 Euros	$1 US ≈ ■ Euros	$■ US ≈ 1 Euro
$50 US ≈ 49 Swiss Francs	$1 US ≈ ■ SF	$■ US ≈ 1 SF
$3 US ≈ 2 Pounds (UK)	$1 US ≈ ■ Pounds	$■ US ≈ 1 Pound
$4 US ≈ 5 Singapore Dollars	$1 US ≈ ■ SGD	$■ US ≈ 1 SGD

Note: Exchange rates often change from day to day; there are Web sites that have the most up-to-date exchange rates.

b. How can you use this information to convert euros to Australian dollars or Swiss francs to Singapore dollars? Explain.

Mathematical Reflections 2

In this Investigation, you used ratios to share equally and unequally according to certain rules. You used rate tables and unit rates to solve problems. These questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary in your notebook.

1. a. **How** can you determine a unit rate for a situation?

   b. **Describe** some ways that unit rates are useful.

2. a. **What** strategies do you use to make a rate table?

   b. **Describe** some ways that rate tables are useful.

3. **How** are your strategies for writing equivalent ratios the same as or different from writing equivalent fractions?

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of the Problems. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Jayden described his thoughts in the following way:

> We used rate tables to find the prices for different amounts of chewy fruit worms in Problem 2.3.
>
> In the rate table, we noticed a repeated pattern such as "for every 5 worms we need to pay $.50." Some of us expressed this pattern in the amount of a unit rate: the money per each worm or number of worms per $1.
>
> In figuring out how much we need to pay for 300 worms, we used our rate table and noticed that there is a $3 increase for every 30 worms.
>
> **Common Core Standards for Mathematical Practice**
>
> **MP7** Look for and make use of structure

- What other Mathematical Practices can you identify in Jayden's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.